Article

Mathematical model of dissolved microbial products in sewage treatment system

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Abstract: Water is the source of life, but all kinds of water resources in the world are suffering from different degrees of pollution. Water pollution leads to a serious shortage of available fresh water resources, and sewage treatment is the main way to solve water pollution. For the sewage after biological treatment of activated sludge, the organic matter contained in the effluent is mainly the dissolved microbial products (SMPs) produced in the process of microbial metabolism. The composition of SMPs is complex, mainly including macromolecular substances such as protein, polysaccharide, humic acid and DNA and cell fragments. The mathematical model of activated sludge is a quantitative description of the mathematical relationship between substrate degradation parameters and microbial growth. Starting from the Monod equation representing the relationship between substrate consumption and microbial growth, it combines the reactor theory and microbiology theory in the chemical field. Based on the principle of conservation of materials and Monod equation, the mathematical expression of organic degradation model was determined according to the collected data and empirical values of parameters. The general idea of activated sludge model No.1 (ASM1) for activated sludge process simulation was introduced, and the influence of sludge concentration on SBR process water treatment was explored. It was found that the removal rate of COD, ammonia nitrogen, total nitrogen and total phosphorus increases with the increase of sludge concentration. When C/N=8, the removal rate of ammonia nitrogen increased from 62% to 81%, and the removal rate of total nitrogen increased from 64% to 82%, with the most obvious effect.

Keywords: dissolved microbial products; mathematical model; sewage treatment system; water resources

1. Introduction

Water is the most important material basis for all life on earth. Although about 75% of the earth's surface is covered by water, 96.53% of it is distributed in the ocean, while the freshwater resources on land only account for 2.53% of the total. With the rapid growth of world population and economy, on the one hand, the demand for fresh water resources is increasing; on the other hand, the discharge of industrial and agricultural wastewater pollutes water bodies more and more seriously. Water pollution further reduces the amount of available fresh water and aggravates the water shortage. The shortage of fresh water resources and water pollution have become an important factor that seriously restricts the economic development of China and the world.

For a long time, biochemical wastewater treatment has been an important water treatment technology, which has unique advantages in technology, economy and

environment, and plays an increasingly important role in municipal and industrial wastewater treatment, advanced treatment and recycling. In recent years, wastewater treatment has become more and more complicated. Traditional physical and chemical treatment methods can no longer meet the needs of industry, and biochemical treatment technology has been introduced into the wastewater treatment process. The decomposition of bacterial organics further reduces the organic pollutants in the wastewater, so that the wastewater can reach the standard of re-discharge or discharge to protect the environment. Microbes have the advantages of wide distribution, rapid reproduction and strong adaptability, and have been paid more and more attention in wastewater treatment. The introduction of biochemical methods in advanced wastewater treatment will be the future development direction. SMPs is the main organic substance in sewage effluent. Its existence not only inhibits the activity of activated sludge and microorganisms in sewage treatment system, and affects the effluent quality of biological treatment, but also can react with disinfectants to produce DBPs with teratogenic, carcinogenic and mutagenic properties, which seriously restricts the regeneration and reuse of sewage. The current sewage treatment system has become a key part of urban infrastructure, aiming to remove harmful substances in wastewater through physical, chemical and biological means to ensure the safety of the water environment. Sewage treatment technology is becoming more and more sophisticated, and treatment standards are becoming increasingly stringent. However, in the face of challenges such as population growth, accelerated industrialization and new pollutants, sewage treatment still faces many technical, economic and environmental pressures.

In this paper, the simulation study of SBR process under different process parameters was carried out, and SMPs of wastewater was simulated. The analysis of water quality showed that the COD removal efficiency gradually increased from the initial 79% to 95% with the passage of time within 30 days of domestication, and the final stable removal rate was about 92%. The effects of sludge concentrations of 1000 mg/L, 1500 mg/L, 2000 mg/L, 2500 mg/L and 3000 mg/L on SBR process water quality treatment was analyzed emphatically. The influent water quality of C/N = 2,4,8 was adopted to simulate the effluent water quality of SBR process. After the experiment, it was found that the removal rate of COD, ammonia nitrogen, total nitrogen and total phosphorus increased with the increase of sludge concentration. Among them, when the sludge concentration was 1000 mg/L, the removal rates of COD, ammonia nitrogen, total nitrogen and total phosphorus were not high. The main reason was that when the sludge concentration was low, the number of microorganisms was small, which made it difficult to remove a large number of influent nutrients reasonably in the operation cycle of the reactor. It was concluded that when the sludge concentration was between 1500 mg/L and 2500 mg/L, the influent organic matter content was suitable for sludge, and the removal rate was greatly improved.

2. Related work

Contamination of membranes by soluble microbial products (SMP) has been among the limits to the widespread use of membranes over the past two decades, so

that the properties and behavior of SMPs have attracted attention and efforts have been made to elucidate their role in membrane contamination in membrane bioreactors. Yet, to date, remarkably very few studies have been published that are specifically relevant to this field. The purpose of the earlier reviewing was to focus on SMP and its effects on bioreactor processing plants and their effluents. Shi [1] has only briefly reviewed the assessment of SMP-induced membrane contamination at the local level (key components, subcomponents, hydrophilic and hydrophobic components of SMP) and at the overall level (overall functions, characteristics and factors of SMP), which will significantly contribute to enable investigators and engineers to better appreciate the practical contributing and take effective measures to avoid the contamination caused by SMP in MBR. Zhang [2] investigated the effect of zinc oxide nanoparticles (ZnO NP) on the hypoxic-aerobic immersed membrane bioreactor (MBR) performance and the characteristics of soluble microbial products (SMPs) produced in the presence of ZnO NP. The removal of NH_4 single bonds was reduced by 8.1% and 21.1% at ZnO NP concentrations of 10 mg/L and 50 mg/L respectively. The SMP concentrations increased by 12.8%, 42.4% and 51.5% after exposure to 1 mg/L, 10 mg/L and 50 mg/L ZnO NP respectively. Size exclusion chromatography (HP-SEC) analysis showed that the presence of ZnO NP at concentrations of 1 mg/L and 10 mg/L resulted in a significant increase in high molecular weight (MW) (583 kDa) SMPs. At the end of the experiment, a significant decrease in the concentration of high molecular weight compounds in the MBR effluent was observed. Excitation-emission matrix (EEM) fluorescence mapping revealed that SMPs consisted mainly of amino acids, tryptophan proteins, polycyclic aromatic hydrocarbons and polycarboxylic acids. This work may contribute to a better understanding of the effects of nanoparticle exposure on wastewater treatment performance and SMP properties. Dong [3]evaluated the characteristics of soluble microbial products (SMP) during denitrification in an aerobic granular sludge system at different chemical oxygen demand (C/N) ratios. The excitation-emission matrix identified four peaks in SMP, consisting of fragrant PN-like, chromogenic PN-like, flavic acid-like, and histidine like substances. Fluorescence region integration showed that the percentage of biodegradable PN-like substances in SMP ranged from 53.0% to 61.7%. The combination of simultaneous fluorescence spectroscopy and two-dimensional correlation spectroscopy showed that the release of SMP fractions in the early stages (0 min–150 min) varied in the following order: PN-like fraction > xanthate-like fraction. Membrane contamination is a major problem in membrane bioreactors (MBR). Cao [4] investigated mine contamination induced by retaining and sorption of membranes. Filtration experiments showed that membranes retained SMP, leading to membrane pore clogging and gel layer formation. Interval adsorption experiments showed that the adsorption of SMP by polyvinylidene fluoride membranes is a spontaneous physical adsorption process. Also, the absolute value of ΔG for SMP adsorption by D3520 was higher than that of PVDF membrane, so SMP was preferentially adsorbed on D3520 rather than PVDF membrane. Therefore, the effect of ARs on reducing the SMP concentration was investigated. The result showed that g of 6D3520 is suitable for SMP adsorption. This physical adsorption includes outer membrane diffusion, inner particle diffusion and surface adsorption. The Redlich-Peterson isotherm model is the best model to describe this

equilibrium data. The bioreactor simulation system validated the membrane contamination mitigation mechanism. An example study was performed for the AR-MBR system. The results showed that the addition of D3520 can mitigate the progression of membrane fouling significantly. The study of soluble microbial products was an important research direction for sewage treatment, but the description of the process was not clear because the mathematical model was not introduced to simulate sewage treatment.

Since the 1980s, installed reactivated sludge method mathematical models have been recognized and widely adopted by retailers. Software applications for modeling and simulation of activated sludge plants were also numerous, but the actual usage of these tasks was quite limiting. A primary reason for this situation was the difficulty of the model calibration process, which required the collection of extended data sets at the investigated plants. These data were usually not included in formal plant maintenance plans. Andraka et al. [5] discussed the problem of using data sets from standard monitoring programs for model calibration, especially considering simulation objectives and data availability. The study was based on operation figures at the Bialystok sewage processing facility. Models for this facility were developed upon the Reactive Sludge Model #3 developed by the IWA Working Group and realised in the ASIM emulator. The model alignment and qualification gave hopeful outcomes; however, careful consideration for further implementation should be given, mostly because of the uncertainty of the input data. Under simulated summer high temperature stress, Tian-Wei [6] studied the effects of temperature (30 °C–45 °C) and ammonia nitrogen volume load on nitrification function and microbial community of activated sludge in aerobic pool of sewage treatment plant. At the same time, under the impact of high temperature, the bioaugmentation effect of medium-temperature concentrated nitrification sludge (with or without domestication) was evaluated in two biological treatment systems. The results showed that at 30 °C–40 °C, the removal rate of ammonia nitrogen (NH^{4+}-N) and the content of nitrifying bacteria by aerobic activated sludge reached over 90% and 4.55%, respectively, and decreased to 40% and 1.97% at 45 °C. In the bioaugmentation test, the reactor inoculated with 5% (volume fraction) domesticated nitrification sludge removed 10% NH^{4+}-N, while the reactor needed to inoculate 10% (volume fraction) medium temperature concentrated sludge to achieve the same removal efficiency. The results showed that the medium-temperature enhanced nitrification sludge acclimated at 40 °C had a good enhancement effect under high-temperature impact load. Feng [7] developed a new mathematical model that includes the kinetics and simultaneous storage and growth of biopolymers for the treatment of high quality and energy efficient low carbon source wastewater. An initial set of parameter values was specified as a combination of estimated, literature and fitted values to effectively simulate a cyclic activated sludge technology (CAST) system. The calibrated model had good performance compared to the experimental data of the CAST system. The model suggested that the recommended condition for CAST dosing of low carbon source wastewater was a volume ratio of 7/28 between the anoxic and aerobic zones. In addition, the use of high-throughput 16S rRNA gene sequencing not only described the microbial community in CAST reactors operating at two feed ratios, but also indirectly validated the model predictions. These methods

have provided some data for our research, but they have not been recognized by the public because of the short research time and small sample size.

3. Discussion on method of mathematical model of dissolved microbial products in sewage treatment system.

3.1. Sewage treatment

Sewage treatment is an important part of water cycle. Anaerobic wastewater treatment technology is widely used in wastewater treatment because of its low energy consumption, high treatment efficiency, and low sludge yield and energy recovery. Anaerobic wastewater treatment produces biogas, including trace phosphine, but also more gases, such as hydrogen, methane and carbon dioxide [8]. It is important for the biogeochemical cycle of phosphorus. Hydrogen is a good energy carrier with high potential recovery value, while methane and carbon dioxide are important greenhouse gases. Therefore, a comprehensive understanding of biogas production in anaerobic wastewater treatment system is very important for understanding biogeochemical phosphorus cycle, energy gas production and the stability of climate system [9].

According to the four-stage theory of anaerobic digestion, the anaerobic digestion process includes four stages, namely, hydrolysis and fermentation stage, hydrogen and acetic acid production stage, methane production stage and homoacetylation stage [10]. As shown in **Figure 1**, in the process of anaerobic digestion, complex organic compounds are first hydrolyzed and fermented into fatty acids and alcohol by fermenting bacteria, then converted into acetic acid and H_2/CO_2 by hydrogen-producing and acetic acid-producing bacteria, and so on, and finally converted into CH_4 by methane-producing bacteria [11]. In anaerobic wastewater treatment, CH_4 produced by acetic acid decomposition accounts for about 70% of total CH_4 production, and CH_4 produced by H_2/CO_2 conversion accounts for about 30% of total CH4 production. H2/CO_2 produced by hydrogen and acetic acid can be formed into acetic acid by acetobacter isotype. However, the amount of acetic acid produced by this process is relatively small, generally accounting for only 5% of acetic acid sources in anaerobic systems [12].
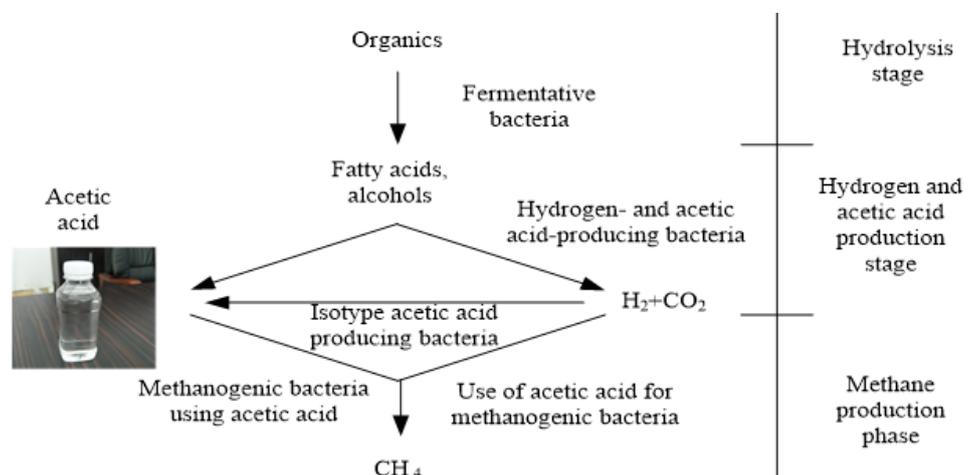

Figure 1. Four-stage theory of anaerobic digestion.

With the increasing scarcity of water resources around the world, wastewater has become an important part of water supply. In many areas, wastewater from treatment plants is discharged into surface water such as rivers, lakes and reservoirs, and reused as water. Even if the sewage from the treatment plant reaches the discharge standard, they still contain some organic substances due to the limited treatment capacity of the treatment plant [13]. Biological treatment will produce soluble microbial products (SMPs), which is inevitable and constitutes most of the organic matter. SMPs can react with chlorine, increase the content of disinfection by-products (DBPs) in water, and make sewage reuse have certain ecological risks [14].

SMPs play an important role in the wastewater treatment process. Their complex composition includes proteins, polysaccharides, humic acid and DNA. Proteins are derived from microbial metabolism and cell decomposition. They can enhance the flocculation of sludge. At the same time, they may also deposit on the membrane surface, causing membrane fouling. Polysaccharides are important components of microbial extracellular polymers. They have good adhesion, promote sludge sedimentation, and form a fouling layer on the membrane surface, further reducing the flux of the membrane. Humic acid is a product of organic matter degradation. It can form complexes with metal ions, affecting the removal of pollutants in wastewater. At the same time, it may also deposit on the membrane surface and increase membrane resistance. The complex composition of SMPs not only affects the efficiency of wastewater treatment, but also has an important impact on membrane fouling and stability. Understanding the specific mechanism of action of these components can provide a theoretical basis for the optimization of wastewater treatment processes, and also help to design more effective membrane cleaning and maintenance strategies [15–16].

SMPs is an organic compound released into the solution by microorganisms through substrate metabolism and biomass degradation, and it is an inevitable product in the process of sewage biological treatment [17]. SMPs is the most important organic matter in sewage effluent, and the dissolved organic matter (DOM) in biological treatment effluent is actually SMPs. SMPs directly determines the concentration of Chemical Oxygen Demand (COD) in effluent, and 83%–91% of COD in effluent comes from SMPs [18].

According to the growth stage of bacteria, SMPs can be divided into two categories: one is substrate utilization related products (UAP), the production of which is related to substrate metabolism and biomass increase, and its production rate is proportional to substrate utilization; the other is biomass-related products (BAP), the generation of which is related to biomass degradation, and its generation rate is directly proportional to biomass concentration [19]. It is pointed out that UAP is mainly a small molecular carbon-containing compound produced by substrate degradation, while BAP is mainly a cellular macromolecular compound, which contains both carbon-containing compounds and nitrogen-containing compounds. The generation process of SMPs is shown in **Figure 2** [20].
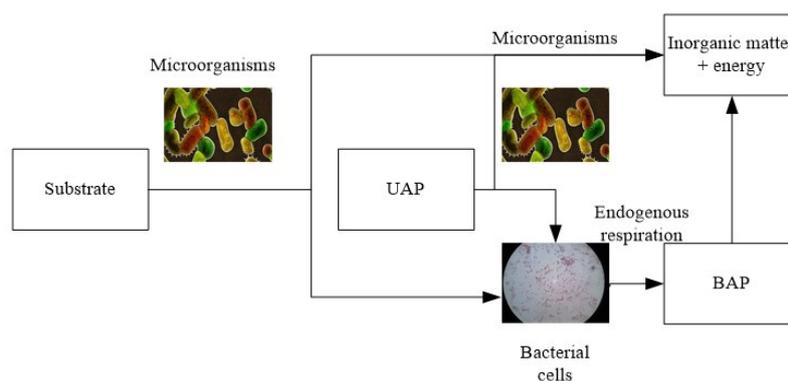
Figure 2. Generation process of SMPs.

The analysis of SMP shows that protein, lipids, polysaccharides and nucleic acids are good anionic ligands, which contain some complex functional groups, such as carboxyl, hydroxyl, mercapto and phenolic groups, and can combine with organic substances or metal ions through hydrogen bonding or chelation, thus affecting the potential toxicity and bioavailability of metals [21]. On the one hand, the chelation of SMP with metals reduces the toxicity of metals; on the other hand, due to the special requirements of anaerobic microorganisms for some trace metals (Fe, C_0, N_i), the chelation of SMP may affect its bioavailability [22].

The existence of SMP hinders the further reduction of COD in sewage, and because of the toxicity of SMP, it will lead to membrane pollution and disinfection by-products [23]. The treatment of SMP is not only related to whether it meets the sewage standard, but also affects the sanitation and human health after sewage discharge. At present, SMP control is mainly divided into source control and pipeline end control. Membrane fouling mainly refers to the deposition of substances on the membrane surface and in the pores, which affects the flux and separation performance of the membrane. It mainly manifests itself in three aspects. First, SMP can gather on the membrane surface to form a biofilm, which can hinder water flow and reduce the flux of the membrane. Secondly, SMP will combine with sludge particles, increase the viscosity and adhesion of the sludge, make it easier to adhere to the membrane surface, and cause rapid fouling of the membrane. Finally, the composition and properties of SMP can change the physical and chemical properties of the membrane surface (such as surface charge and hydrophilicity), further affecting the fouling behavior of the membrane.

3.2. Mechanism of biodegradation

The decomposition of microbial organic matter is a process in which macromolecular organic matter is decomposed into micromolecule organic matter or simple inorganic matter through a series of biochemical reactions. According to the type of biodegradation and the final product, the biodegradation of organic matter can be divided into different types, such as biological removal, pre-degradation, environmentally safe degradation and complete degradation [24].

There are two types of microbial degradation: aerobic degradation and anaerobic degradation, both of which contain facultative microorganisms. It should be pointed out that compared with anaerobic microorganisms, aerobic

biodegradation of organic matter usually has the characteristics of higher degradation rate, more complete degradation degree, higher energy utilization rate and higher cell conversion rate, and less strict requirements on environment (such as temperature, pH) [25]. For the metabolites of these two treatment methods, the products of aerobic treatment are basically harmless, and the sewage has no peculiar smell; the products produced by different oxygen treatment are complex and produce water with different odor.

In the process of microbial degradation, there are three interactions: synergy, inhibition and predation. Synergism is a common phenomenon, and it's also what we expect, because the existence of synergy makes it possible to strengthen the degradation and removal of pollutants by using composite microbial communities of mixed media in different environmental fields.

Microbial degradation performance is affected by microbial activity, compound structure, temperature, pH, nutrition and oxygen supply. Among them, two key points are emphasized: (1) The activity of microorganisms is the most important factor, and the logarithmic phase in which microorganisms have the fastest ascending rate is the most vigorous metabolic rate and the strongest activity. The types of microorganisms in the target water body also determine the direction of biodegradation. (2) The structure of chemicals determines the complexity of degradation mechanism, and the nature and quantity of functional groups have great influence on the biodegradation of organic compounds. Among them, molecular weight is better than molecular weight, linear hydrocarbon is better than branched hydrocarbon, and the more branched alkyl groups, the more difficult it is to degrade. When all the hydrogens on carbon atoms are replaced by alkyl or aryl groups, bio-impedance substances will be formed. Toxic substances interfere with the activities of microorganisms to some extent, which may cause paralysis of the whole process in severe cases. Therefore, when treating oilfield sewage by biological method, it is necessary to detect the concentration of toxic substances in sewage, and entering biological method within the allowable concentration range can improve the treatment efficiency.

3.3. Introduction of activated sludge mathematical model (ASM)

With the advantages of good treatment effect and high removal rate, activated sludge method has become the main biological sewage treatment method, which has been widely used all over the world, and the sewage treatment technology has been greatly developed and innovated. However, the sewage treatment experiment takes a lot of time and material resources, which greatly limits the development of sewage treatment technology [26,27]. Activated sludge mathematical model is a process that can describe the biochemical reaction of each component in the reactor in mechanism, and at the same time, it can describe the change of each component in the reactor to predict the water quality. Through the establishment of mathematical model, the operation of sewage treatment plant is analyzed, and the ability of optimal management of water plant is realized through optimization analysis. There is a good description of the process of nitrification and denitrification by oxidizing and nitrifying bacteria of organic matter, and there is a good simulation result in sewage

treatment.

At present, there are a series of mathematical models of activated sludge, such as ASM1, ASM2 and ASM3 models. Each model has a certain scope of application, and has different advantages for different applications. The adopted parameter component theory is also different, and the comparison is shown in **Table 1**.

Table 1. Comparison of ASMs series models.

Model	Theoretical basis	Function	Biochemical reaction process (pcs)	Number of stoichiometry (pcs)	Model components (pcs)
ASM1	Regeneration theory of death	Carbon removal, nitrogen removal	8	5	13
ASM2	Death-regeneration and storage theory	Carbon removal, nitrogen removal, phosphorus removal	19	22	19
ASM3	Endogenous respiration and storage theory	Carbon removal, nitrogen removal	12	22	13

The main feature of activated sludge mathematical model No.1 (ASM1) is that it is based on the mass balance of COD and describes the relationship among concentration, reaction rate, kinetic parameters and stoichiometric coefficient in matrix form. This paper describes eight reactions in wastewater under aerobic and anoxic conditions, including hydrolysis, organic decomposition, microbial growth and attenuation, with five stoichiometric coefficients and 14 kinetic parameters, including 13 kinds of water quality components, which can be divided into two categories: dissolved components (denoted by symbol E) and solid components (denoted by symbol X). In activated sludge system, the biological reaction rate of each component is equal to the sum of the rates of each component in each biological reaction, which can be expressed by mathematical Equation (1).

$$\upsilon_i = \sum_{j=1}^{8} \upsilon_{ij}\rho_j \tag{1}$$

In which: v_i is the biological reaction rate of the i-th component in the ASM1 matrix in the activated sludge system, v_i is the stoichiometric coefficient of the i-th component in the ASM1 matrix in the j-th process, and ρ_j is the mathematical expression of the basic rate of the j-th biological reaction process in ASM1.

Take the following traditional activated sludge process (see **Figure 3**) as an example to illustrate the method and process of establishing material balance equation. In **Figure 3**, t represents the wastewater flow rate; V represents the volume of aeration tank; D represents water quality components (including soluble component E and solid component X); K represents the reflux coefficient; Subscript in indicates sewage inflow term; F indicates the effluent of aeration tank; E represents the outlet item of secondary sedimentation tank; W represents the excess sludge effluent item; R represents the reflux term; It is the sequence number of water quality components in ASM1 matrix.
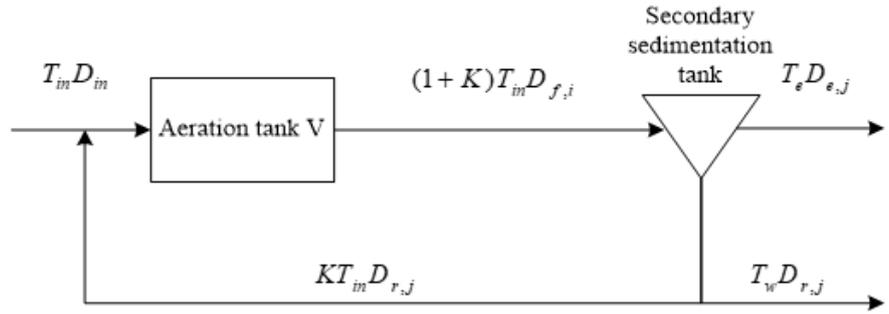
Figure 3. Material balance diagram of aeration tank

All kinds of substances participating in the reaction in the reactor abide by the principle of conservation of materials, that is, the substances change in the reactor, but the total amount of substances remains constant. Material conservation is based on the law of mass conservation, and the material balance equation with the reactor as the boundary is shown in Equation (2).

$$V\frac{dD_i}{dt} = \left[T_{in}D_{in,i} + KT_{in}D_{r,i} - (1+K)T_{in}D_{f,i}\right] + V \cdot r_i \tag{2}$$

For completely mixing the dissolved components in the reactor, if it is assumed that

$$E_{r,i} = E_{f,i} \tag{3}$$

The Equation (2) is modified to the Equation (4).

$$V\frac{dE_i}{dt} = \left(E_{in,i} - E_{f,i}\right)T_{in} + V \cdot r_i \tag{4}$$

For dissolved oxygen, artificial aeration should be considered in material balance, as shown in Equation (5).

$$V\frac{dE_8}{dt} = \left[T_{in}D_{in,8} + VC_l\left(D_{EW} - E_{f,8}\right) - T_{in}E_{f,8}\right] + V \cdot r_8 \tag{5}$$

where: C_l is the oxygen transfer coefficient in the reactor; D_{EW} is the dissolved oxygen saturation of the mixed solution in the reactor.

The SBR process still does not enter water in the precipitation stage. In this model, it is assumed that it is completely ideal precipitation, and it is considered that standing is the continuation of precipitation and drainage. It is considered that the reaction in the reactor does not occur at this time, and the components do not change. Therefore, the reaction equation at this time is as follows.

Formula of drainage:

$$V_{out} = T_{out} \cdot t \tag{6}$$

3.4. Degradation rate of organic matter

The growth of heterotrophic bacteria is the result of continuous absorption of organic nutrient elements in sewage. It can be seen that there is some quantitative

relationship between the growth of bacteria and the degradation of organic matter. The microbial productivity W indicates the number of microorganisms increased by using the unit substrate concentration, namely:

$$W = dX \big/ dE \tag{7}$$

Among them, dE indicates the amount of organic matter used by bacteria; dX Indicates the increase of bacteria after utilize dE.

The degradation rate of organic matter is defined as the change of organic matter concentration in dt time period, which is represented by V. V is a vector. As the concentration of organic matter decreases with time, V is negative. The mathematical expression is:

$$v = -\frac{dE}{dt} \tag{8}$$

Substituting Equation (7) into Equation (8) can obtain the degradation rate V of organic matter as follows:

$$W = \frac{dX}{dE} \Rightarrow dE = \frac{dX}{W} \tag{9}$$

$$v = -\frac{dE}{dt} = -\frac{1}{W}\frac{dX}{dt} \tag{10}$$

$$\frac{dX}{dt} = \lambda X \tag{11}$$

$$v = -\frac{1}{W} \cdot \lambda X \tag{12}$$

In the Monod formula:

$$\lambda = \lambda_{\max} \frac{E}{C_E + E} \tag{13}$$

$$v = -\frac{1}{W} \cdot X \cdot \lambda_{\max} \frac{E}{C_E + E} \tag{14}$$

Among them, W represents the growth rate coefficient of heterotrophic bacteria; X represents the bacterial concentration; C_E represents the maximum specific growth rate of bacteria; C_E represents the semi-saturation constant, and C_E is when $\lambda = \frac{\lambda_{max}}{2}$, the concentration e of organic matter.

When the substrate concentration E is very low, the formula can be simplified as:

$$v \approx -\frac{\lambda_{\max}}{C_E} \cdot \frac{X}{W} E \tag{15}$$

Low substrate concentration is generally defined as S < 500 g/mL, and the field data collected in this study meet this requirement, so the degradation rate of organic matter is shown in Equation (15).

3.5. Principle and characteristics of SBR process

Sequencing Batch Reactor (SBR), also known as batch sequencing bioreactor, is the gradual development of filling and discharging reactor. This technology is widely used in wastewater treatment in recent years. The basic sequence of operation is as follows: water inlet stage, reaction stage, precipitation stage, dehydration stage and waiting stage, as shown in **Figure 4**.
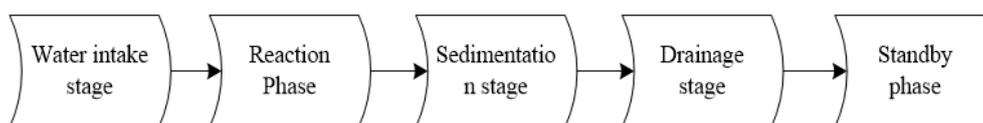

Figure 4. Operation of SBR process.

The operation of SBR process is characterized by intermittent operation. Intermittent formula has two meanings: first, it means that the operation is consistent in space and intermittent. However, because wastewater discharge is mostly continuous, SBR usually uses two or more reactors to run simultaneously. Another conclusion is that time is sequential and discontinuous. The whole operation process is divided into five stages. Inflow → reaction → precipitation → discharge → standing, which represents a complete cycle.

4. Experimental design of mathematical model of soluble microbial products

4.1. Generation of SMPS

SMPs of simulated wastewater: A simple sequencing batch reactor (SBR) was used to produce SMPs (**Figure 5**). Activated sludge comes from the aeration tank of a sewage treatment plant in a city in China. Sludge concentration is about 8000 mg/L–9000 mg/L. In laboratory culture, the initial color is dark brown, with slight odor, the sludge has poor sedimentation, and it is difficult to settle completely for a period of time. Besides, there are suspended substances in the upper layer in the clear night, and the water quality is turbid, which indicates that the sludge state is not the best, so domestication and cultivation are needed to meet the experimental requirements. For 48 h, the floating materials on the surface of the initial sludge discharge and the large sediments at the bottom of the lab setup were aerated without any external input to deplete the residual nutrients in the activated sludge. After this 48-hour period, the sludge was inoculated into an SBR reactor, and its concentration was diluted to maintain approximately 3500 mg/L. Artificial domestic wastewater was then used for cultivation and acclimation over a period of 25 to 30 days.
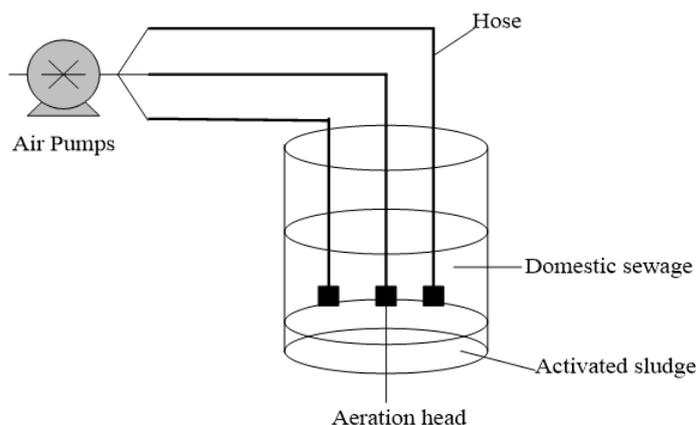
Figure 5. Experimental device diagram.

Table 2 for the value range of heterotrophic bacteria reaction rate constant.

Table 2. Reaction rate constant of denitrification process.

	Reaction rate constant	Unit	Value
Maximum specific growth rate of heterotrophic bacteria	λ_{max}	d-1	3.1–12.9
Organic saturation constant	C_E	mg/L	11–175
Maximum yield coefficient	W_{max}	g/g	0.67g cell COD/g Matrix cod
Attenuation constant	b	d-1	0.05–0.1

Table 3 shows the values of each parameter in the degradation model. Among them, the reciprocal of the ratio of the influent flow of sewage system to the effective volume of the reaction tank is the hydraulic retention time.

Table 3. Parameter values in degradation model.

Model Parameters	Take value	Model Parameters	Take value
Dissimilar bacteria ratio growth rate λ	3d-1	Heterotrophic bacteria yield coefficient W	0.67 mg/mg
Half-saturation constants	150 mg/L	Hydraulic residence time HRT	6 h
MLVSS	1600 mg/L	$\frac{T}{V}$	3–6 d-1

4.2. Experimental design of sludge concentration on sbr process water quality treatment

Sludge is an aggregate of microorganisms, and its concentration represents the number of microorganisms in the reactor. Generally speaking, the higher the sludge concentration, the more microorganisms in the reactor. The larger the number of microorganisms, the greater the demand for organic matters, the higher the utilization rate of organic matters in sewage entering the reactor, and the better the treatment effect. In this paper, the influence of sludge concentration on the sewage treatment effect of SBR reactor was explored from the angle of mathematical model and simulation.

In this paper, a cycle was 6 h, the reaction temperature was 25 ℃, the ratio of

flushing was 2/8, the water was fed for 15 min, the stirring was started at the same time, the stirring time is 5 h, the anoxic time was 1 h, the aeration was started after the anoxic time was finished, the aeration time was 4 h, and the final hour was left for precipitation, and the water was drained after the precipitation was finished.

5. Discussion on experimental results of mathematical model of dissolved microbial products.

5.1. Water quality treatment effect

SBR belongs to batch reactor, and the whole process includes several stages, such as water feeding, stirring, aeration, sedimentation, drainage and standing. The water inlet of this experimental reactor is in the lower part of the reactor, and the peristaltic pump is used to feed water, while the water outlet is in the middle of the reactor. The experimental water inlet is automatically controlled. In the start-up stage, an operation cycle is 6 h, it not only ensures the adequacy of microbial reaction, but also ensures the stability and treatment efficiency of the system. This time arrangement can provide sufficient time for organic matter degradation, nitrification and denitrification reactions, while allowing sludge to settle sufficiently, adapt to the water inlet and outlet conditions set in the experiment, and ensure the continuous and stable operation of the system. The water inlet time is 35 min, the anoxic stirring time is 60 min, the aeration time is 3 h, and the settling time is 1h. The flushing ratio is 1/4 in each cycle, with 7 L water intake and 7 L water drainage. In the initial stage of domestication, the concentrations of CODcr, ammonia nitrogen and total phosphorus in raw water are 200 mg/L–300 mg/L, 30 mg/L–40 mg/L and 6 mg/L–10 mg/L, respectively. Under the experimental conditions, after about 30 days of cultivation and domestication, the concentrations of CODcr, ammonia nitrogen and total phosphorus in the effluent gradually tend to be stable, and the water quality treatment effect is shown in **Figure 6**.


(a)	**(b)**
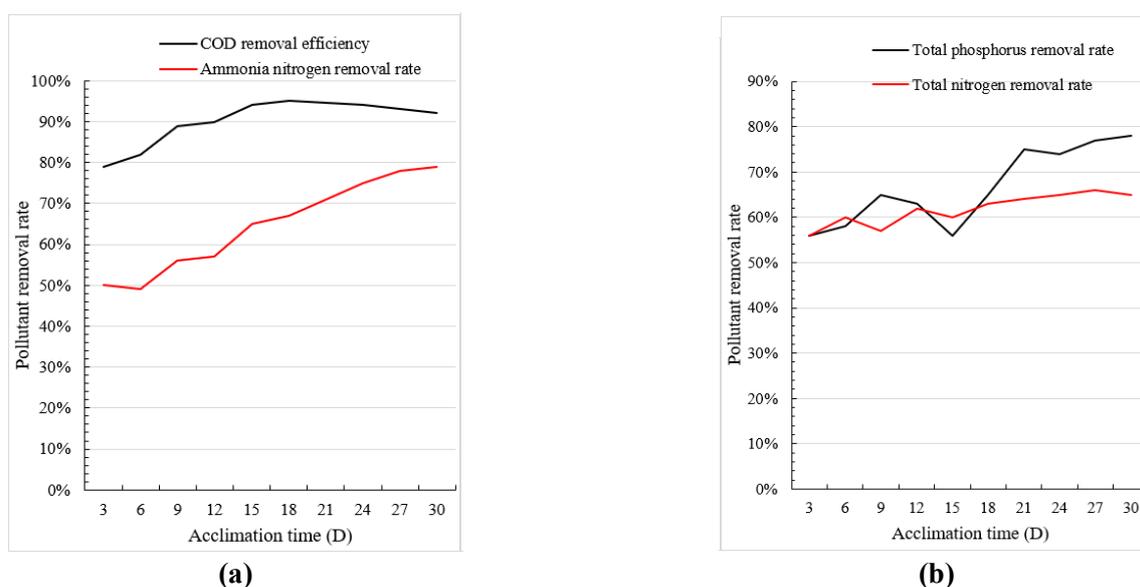
Figure 6. Water quality treatment effect **(a)** The removal efficiency of COD and Ammonia nitrogen; **(b)** The removal rate of total nitrogen and total phosphorus.

As shown in the figure, within 30 days of domestication, the removal efficiency of COD gradually increased from the initial 79% to 95% with the passage of time, and the final stable removal rate was about 92%; the removal rate of ammonia nitrogen increased from the initial 50% to 79% and gradually stabilized. The removal rate of total nitrogen increased from the initial 56% to about 65% with the increase of acclimation time. The removal rate of total phosphorus increased from 56% to 78%. It can be inferred that the sludge in the initial stage of cultivation is pulled back from the sewage plant to the laboratory, and the sludge treatment capacity is poor, and the effluent quality is not up to standard. The main reason is that the sludge is transported back in winter when the temperature is low, and the temperature of transporting back to the laboratory increases, so that the treatment capacity of microorganisms is gradually restored. At the same time, with the extension of training time, microorganisms gradually adapt to the new environment, forming enzymes to treat the corresponding water quality, and improving the treatment capacity. With the domestication, the removal rate of each index gradually increases, and each index is stable, reaching the requirements of effluent quality, which indicates that the sludge treatment capacity at this time is up to the required standard. At this time, it can be considered that the sludge domestication is completed and the start-up work is completed.
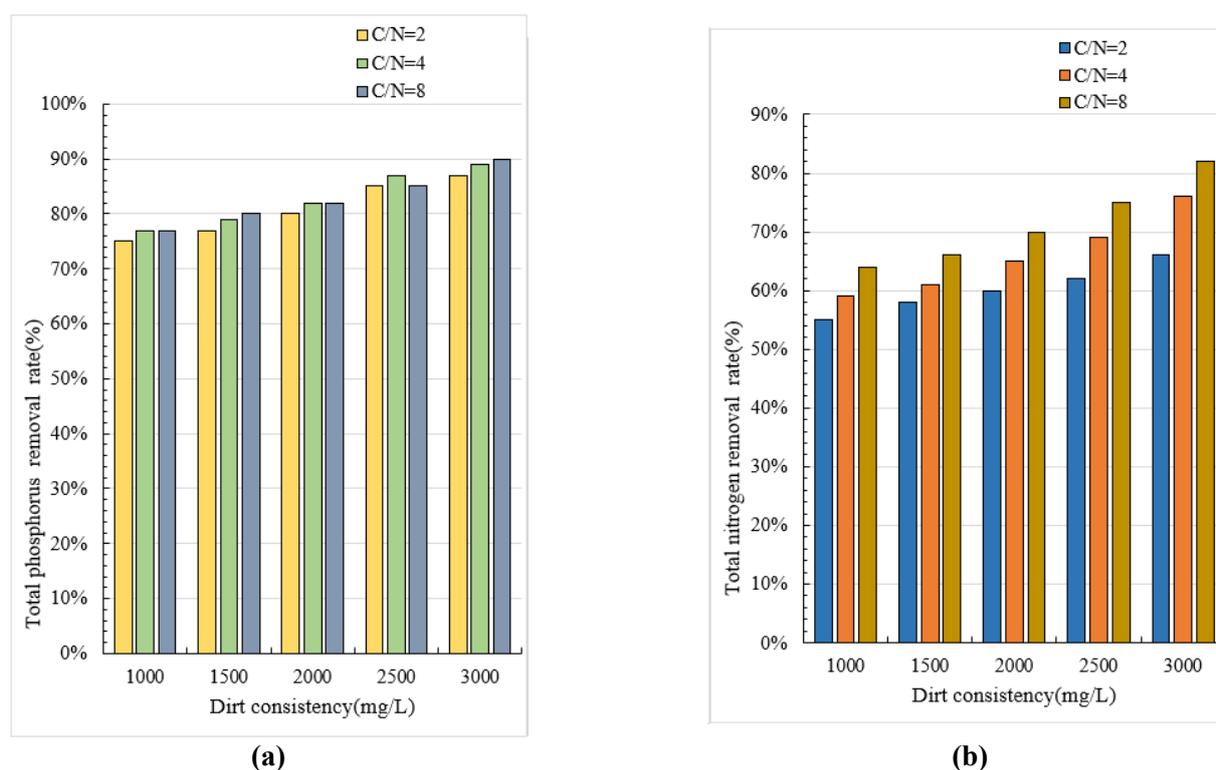
5.2. Influence of sludge concentration on SBR process water quality treatment

In this paper, the influent water quality of SBR was simulated with C/N = 2, 4, 8 when the sludge concentration was 1000 mg/L, 1500 mg/L, 2000 mg/L, 2500 mg/L and 3000 mg/L.

The simulation results of sludge concentration are shown in **Figures 7** and **8**.

(a)   **(b)**
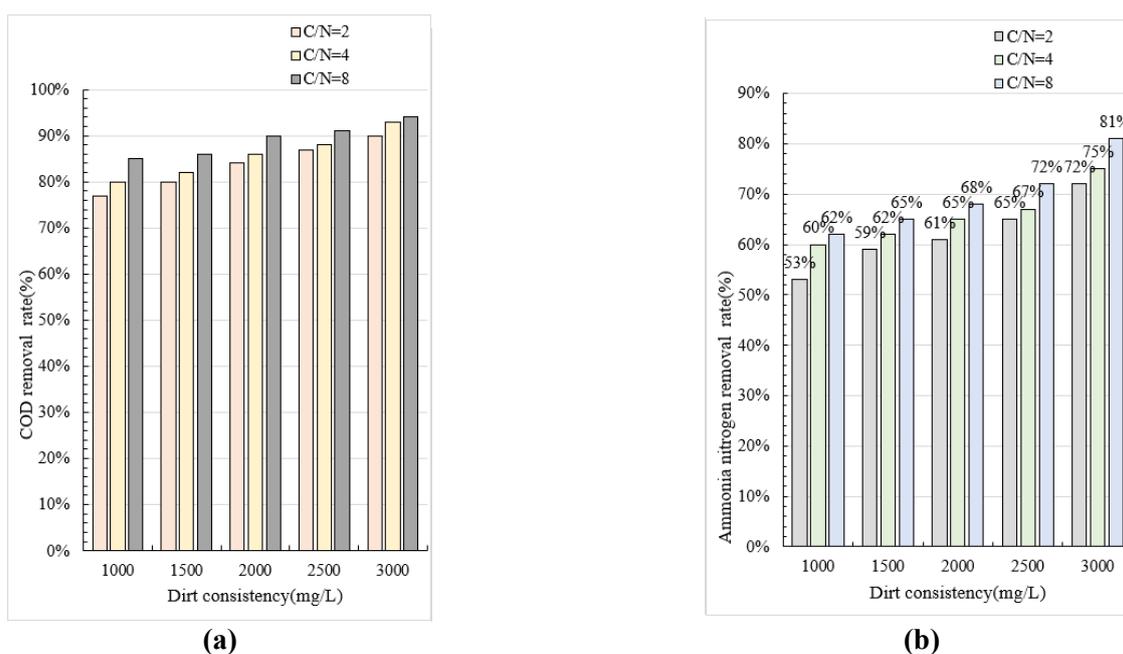
Figure 7. Effect of sludge concentration on removal of COD and ammonia nitrogen at different influent C/N ratio **(a)** The effect of sludge concentration on removal of COD under different influent C/N ratio; **(b)** The effect of sludge concentration on removal of ammonia nitrogen under different influent C/N ratio.

(a)

(b)

Figure 8. Effect of sludge concentration on removal of total phosphorus and total nitrogen at different influent C/N ratio **(a)** The effect of sludge concentration on removal of total phosphorus under different influent C/N ratio; **(b)**The effect of sludge concentration on removal of total nitrogen under different influent C/N ratio.

It can be seen from **Figures 7** and **8** that the removal rates of COD, ammonia nitrogen, total nitrogen and total phosphorus increased with the increase of sludge concentration. The rates of removing ammonia nitrogen and total nitrogen were more obvious with the increase of sludge concentration. When C/N = 8, the removal rates of ammonia nitrogen and total nitrogen increased from 62% to 81% and 64% to 82% respectively. Among them, the removal rates of COD, ammonia nitrogen, total nitrogen and total phosphorus were not high when the sludge concentration was 1000 mg/L. The main reasons for the low removal rate of the SBR system at low sludge concentration are insufficient microbial numbers, decreased metabolic activity, short sludge retention time, poor sludge flocculation and sedimentation performance, low mass transfer efficiency, and imbalanced nutrient ratios. These problems have a comprehensive impact on the degradation capacity of microorganisms in the system and the stability of the system, resulting in a decrease in pollutant removal rate. However, the high sludge load makes the intake of nutrients by microorganisms supersaturated, which leads to the unreasonable removal of a large number of influent nutrients in the operation cycle of the reactor. With the increase of sludge concentration, the number of microorganisms increases, and the demand for organic matters is greater. When the sludge concentration is between 1500mg/L and 2500 mg/L, the influent organic matter content is more suitable for sludge, and the removal rate is greatly improved. When the sludge concentration is greater than 2500 mg/L. The concentration of organic matter is relatively insufficient, the growth of microorganisms is inhibited, and the removal effect increases slowly.

6. Conclusions

In recent years, China has vigorously developed the environmental protection industry, and sewage treatment has been highly valued. At present, many cities in China have set up sewage treatment plants. The wastewater from the biological treatment plant contains many soluble organic compounds, including biodegradable and non-biodegradable residual components in the influent matrix, as well as intermediate and final biodegradable products. After complex biochemical reactions, various organic compounds are formed. SMPs is an important component of organic matter in biological sewage treatment plants, and the mathematical model of activated sludge is an effective tool to study the operation and optimal management of urban sewage treatment plants. In this paper, the influence of sludge concentration on water quality treatment in SBR process was analyzed, and it was concluded that when the sludge concentration was between 1500 mg/L and 2500 mg/L, the influent organic matter content was more suitable for sludge, and the removal rate was greatly improved. Its application in the daily operation and management of sewage treatment plants has effectively ensured the sewage treatment plants to reach the standard, ran stably and reduced the operation cost. Of course, this article also has certain shortcomings. When discussing the complexity and mechanism of action of SMPs, there is a lack of in-depth quantitative analysis, and it is impossible to fully explain the impact of SMPs on membrane fouling. In subsequent studies, more in-depth quantitative analysis will be carried out, combined with intelligent technology to ensure the feasibility and temperature resistance of the results.

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